



# Modeling and Simulation

## Lecture 2

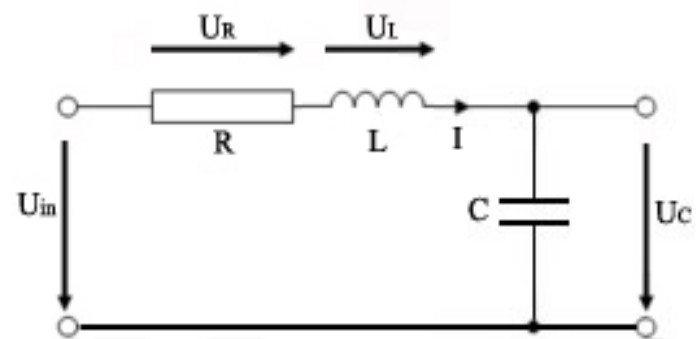
### The Mathematical Model of a Circuit

# Objectives

- Determine the **Mathematical Model** for a 1<sup>st</sup> and 2<sup>nd</sup> order circuit
  - Simulate the circuit functionality starting from the MM
  - Examples
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## Determine the Mathematical Model for a 1<sup>st</sup> and 2<sup>nd</sup> order circuit

- The description of the circuit behavior is based on a *mathematical model* (sometimes called *the circuit model*) which shows the relation between the input and output variables.
- The model describes the evolution of the output variable as a function of the input variable.
- By inspecting the circuit and detecting how many **energy storing components** (inductors and capacitors) it contains, we can determine the order of the system.
- An RLC filtering circuit contains two energy storing components. An inductor and a capacitor. This circuit is of 2<sup>nd</sup> order. The **MM** which describes this circuit will be of 2<sup>nd</sup> order.



## Determine the Mathematical Model for a 1<sup>st</sup> and 2<sup>nd</sup> order circuit

### Accepted assumptions when solving the M&S problems:

- *Electrical effects happen instantly in the entire circuit:* effects induced by electrical signals are present at any moment, at any point in the circuit.
- *There is no magnetic coupling between the system components.*
- *Circuit components are considered ideal.* Because of this assumption an ideal circuit component presents the following features:
  - *It uses only two terminals for connection.*
  - *From a mathematical point of view it is described in terms of current and voltage.*
  - *It can not be further divided into several other components.*

## Determine the Mathematical Model for a 1<sup>st</sup> and 2<sup>nd</sup> order circuit

For a given circuit we will be interested in:

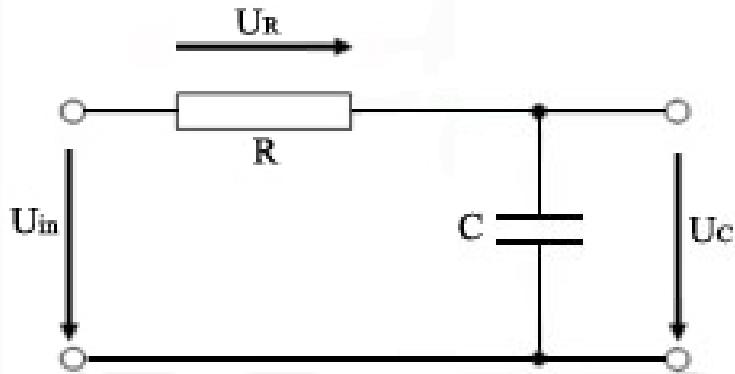
- *voltage* and *current* calculations.
- *power* and *energy calculations*.

The circuit components which will be used can be divided into *passive* or *active* elements. Passive elements can not generate energy while active elements can.

Passive elements are resistors, inductors and capacitors. Sources are active elements and in the ideal case they provide voltage (or current) and are independent from other circuit elements.

Voltage (or current) controlled sources will not be used throughout this material.

## Determine the Mathematical Model for a 1<sup>st</sup> and 2<sup>nd</sup> order circuit

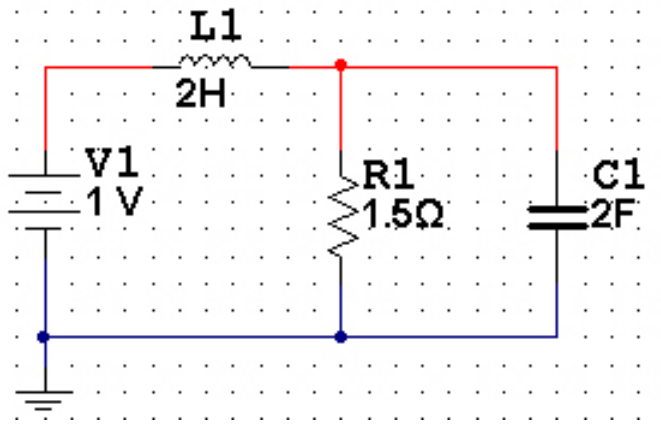


RC series filtering circuit

### For the 1<sup>st</sup> order circuit:

- determine the evolution of the capacitor voltage for the first 10 seconds of the circuit functionality. The filter has zero initial conditions and it receives at the input a step signal with 1V amplitude. The step signal switches two seconds after the start of the simulation. The component values are  $R = 10 \Omega$  and  $C = 50 \text{ mF}$ .
- same problem but the circuit presents 0.3 V initial conditions.

## Determine the Mathematical Model for a 1<sup>st</sup> and 2<sup>nd</sup> order circuit



RLC parallel filtering circuit

### For the 2<sup>nd</sup> order circuit:

- determine the mathematical model of the circuit. Create a program which simulates the step response of the circuit. There are zero initial conditions and the component's values are  $R = 1.5 \Omega$ ,  $L=2 \text{ H}$  and  $C=2 \text{ F}$ . The simulation lasts 100 seconds.
- check the results with the RLC Circuit Analysis tool.